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**RESEARCH ON ESTABLISHING THEORY OF COMBINED
ADJUSTMENT OF SPATIAL TERRESTRIAL GEODETIC AND
GNSS NETWORKS IN ENGINEERING CONSTRUCTION**

MAJOR: SURVEYING AND MAPPING ENGINEERING
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SUMMARY OF THE DOCTORAL THESIS

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The thesis has been completed at the **Department of Engineering Surveying, Faculty of Geomatics and Land Administration, Hanoi University of Mining and Geology**

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INTRODUCTION

1. The necessity of the thesis

The spatial networks were previously built with traditional measurement technology, including measurements of slope distance, horizontal angle and vertical angle. Currently, the spatial networks can add other measurements by GNSS. Research the theory of adjustment of spatial terrestrial geodetic - GNSS networks is significant in the construction of work, especially for high-rise projects that require high precision being widely constructed in our country.

2. Research purpose, object, and scope

The purpose of this thesis is to build the method and algorithm for adjustment of spatial terrestrial geodetic - GNSS networks in order to improve the efficiency for the construction of high accuracy works. The subject of the study is the spatial terrestrial geodetic - GNSS networks. The scope of the research is used for the construction of great heigh different projects.

3. Research contents

Researching algorithm of combined adjustment of spatial terrestrial - GNSS short distance networks; selecting suitable reference system, an algorithm to process data of spatial terrestrial geodetic - GNSS networks and convert to construction coordinates to survey in construction of high rise projects that require high precision; studying on the applicability this networks in construction of great heigh different projects that require high precision.

4. Research methodology

Statistical method, analytical method, empirical method, comparative method, mathematical method, computing application method.

5. The scientific and practical significance of the thesis

The thesis contributes to perfecting the theory of data processing of the spatial terrestrial geodetic - GNSS networks in construction. Research results can be applied in teaching, scientific studies and practice of production.

6. Theoretical points to be defended

- *The first theoretical point:* The algorithm of spatial terrestrial geodetic - GNSS networks presented in the thesis is appropriate to process data of short distance networks in the construction of great heigh different projects.

- *The second theoretical point:* Application of spatial terrestrial geodetic - GNSS networks is processed according to the free adjustment algorithm in accordance

with the nature and process of building a control network to staking out axis in the construction of super high rise buildings. The proposed solutions to transferring height to floors due to spatial terrestrial geodetic - GNSS networks are feasible and highly effective.

- *The third theoretical point:* The thesis has clarified the theoretical basis of determining the correcting values of measurements before adjustment and calibration of coordinates after adjustment of the spatial terrestrial geodetic - GNSS short distance networks, in order to improve the accuracy of staking out axis in construction of super high rise buildings.

7. New points of the thesis

1 - Proposing solutions to apply spatial terrestrial geodetic - GNSS short distance networks in construction of great height different works.

2 - Developing an algorithm suitable for processing measurement data of spatial terrestrial - GNSS short distance networks in construction of great height different works.

3 - Proposing the method of determining correcting values of measurements before an adjustment and calibrating the coordinates after adjustment of the spatial terrestrial - GNSS short distance networks, in order to improve the precision of staking out axis during the construction of super high rise works.

8. Structure and contents of the thesis

The thesis consists of three parts: introduction, four chapters, conclusion.

Chapter 1

OVERVIEW OF SURVEYING CONSTRUCTION CONTROL NETWORK AND DATA PROCESSING OF COMBINED ADJUSTMENT OF SPATIAL TERRESTRIAL GEODETIC - GNSS NETWORKS

1.1. Overview of the geodetic construction control network

1.1.1. Characteristics of the geodetic construction control network

1.1.2. Methods to build a geodetic construction control network

Traditional methods, GNSS technology method, combined terrestrial and GNSS measurements method.

1.2. Overview of data processing of terrestrial geodetic - GNSS networks

1.2.1. Literature overview in the world

Currently, terrestrial and GNSS measurements can be combined for adjustment in a common network. The problem is these types of measurements

are from different reference systems. The researchs can be categorized by the content as the following:

1. Research on data processing of terrestrial geodetic - GNSS networks in planes: [38], [45], [50], [51].
2. Research on classic spatial network adjustment before the existence of GNSS: [32], [40], [47].
3. Research on data processing of spatial GNSS network: [35], [43], [48].
4. Research on spatial terrestrial geodetic - GNSS networks: [31], [35], [44], [45], [50], [52], [53].
5. Research on application GNSS measurement networks in construction of works: [33], [34], [36], [41], [42].

1.2.2. Literature overview in Vietnam

Research on the application of GNSS and terrestrial - GNSS networks in the construction of works in our country can be categorized as:

1. Application of GNSS in the construction of works: [8], [9], [12], [13], [15], [22], [23], [24], [26], [28], [29], [30].
2. Research on data processing of GNSS network and terrestrial - GNSS networks: [5], [7], [10], [11], [14], [16], [17], [18], [19], [20], [21].

1.2.3. Domestic and international geodetic data processing software

Surveying data processing software is very important to have the fastest and most accurate calculation results. However, in our country, the software still focuses on simple network forms, which cannot handle complex network types such as spatial terrestrial geodetic - GNSS networks.

1.3. The general assessment of literature overview at Vietnam and abroad, studying the orientation of the thesis

1.3.1. General assessment of literature overview at Vietnam and abroad

1. Data processing has always been an important internal work of surveying. Research on data processing of terrestrial geodetic - GNSS networks is an issue of interest both in Vietnam and abroad.
2. In many construction cases, application both of measurements in establishing the networks will increase feasibility as well as accuracy.
3. The prerequisite in data processing of the adjustment problem of terrestrial – GNSS network is choosing a reference system for calculation. Each method of choosing a reference system will suit a number of different specific cases.

4. With its outstanding advantages, GNSS has participated in most types of surveying works as well as different types of construction works. Each type of construction work has its own characteristics, requirements, and needs to be flexible in construction as well as network data processing.

1.3.2. Limitations

1. In our country, the GNSS is only applied to one. There are still many problems that have not been clarified, especially data processing of GNSS measurements.
2. The control networks for construction of works in our country today are mainly established as a two-dimensional horizontal networks and one-dimension leveling networks. There are not may researches on the applicability of spatial networks in the construction of works.
3. At projects requiring high precision, such as super-high-rise buildings, it is necessary to apply modern technologies to ensure the required accuracy. The issue of data processing for control network types serving these projects needs attention due to their own characteristics.
4. Due to the development of technology, the form of combined terrestrial geodetic and GNSS networks can be applied in some construction cases. There should be further research that provides a synthesis and overall analysis of data processing of terrestrial geodetic - GNSS networks in the construction of works.

1.3.3. Major studying orientations of the thesis

1. Studying the adjustment of spatial terrestrial geodetic - GNSS networks.
2. Choosing the right reference system and algorithm for data processing of terrestrial geodetic and GNSS measurement networks for surveying work in the construction of great heigh different works with required high accuracy.
3. Researching the applicability of terrestrial - GNSS short distance networks in high accuracy constructions. Determining the corrected values before adjustment, calibrating coordinates after adjustment, developing the data processing software to ensure the surveying work in the construction of great heigh different works that require high precision and work progress.

Chapter 2

SPATIAL SHORT DISTANCE NETWORK IN CONSTRUCTION OF HIGH PRECISION WORKS

2.1. Overview of the survey for high rise construction

2.2. Required precision of survey control networks

2.3. Traditional spatial short distance network in construction of high precision works

2.3.1. Classification of control network by distance length

Table 2.1. Classification of distancen by length

Classification	Very short distance	Short distance	Medium distance	Long distance
Length	10^2 m	10^3 m	10^4 m	10^5 m
	10 ÷ 99 m	100 ÷ 999 m	1,000 ÷ 9,999 m	>10,000 m

Under that classification, the construction control networks are usually the short distance and very short distance ones.

2.3.2. Choose appropriate measuring instrument by distance

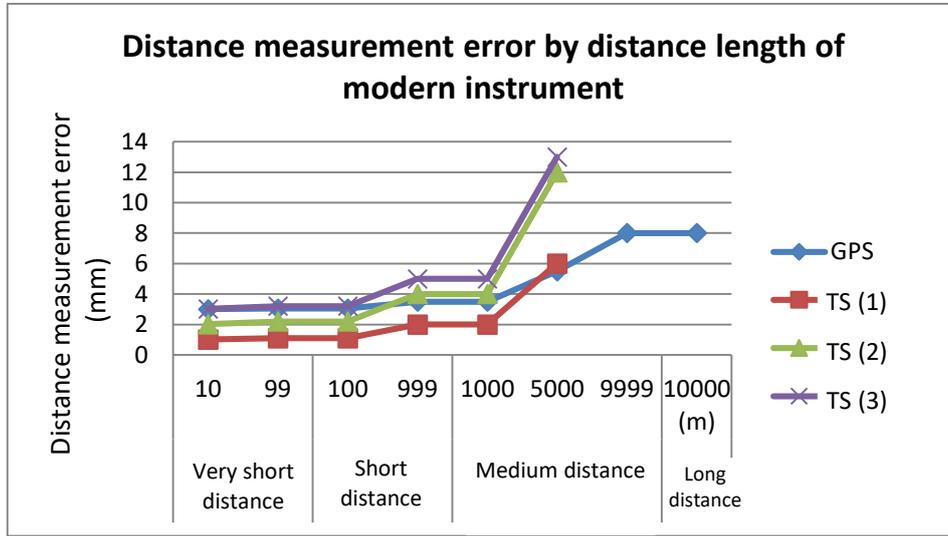


Figure 2.1. Distance measurement error by distance length of the modern measuring instrument

Total station (TS) should be used to measure short and very short distances; GNSS is used to measure long distances, medium distances and combined with TS in measuring short distances.

2.3.3. Process data of traditional short distance spatial network

Residual vector of observations for slope distance S_{ij} :

$$V_{S_{ij}} = -a_{ij} \delta x_i - b_{ij} \delta y_i - c_{ij} \delta z_i + a_{ij} \delta x_j + b_{ij} \delta y_j + c_{ij} \delta z_j + l_{S_{ij}} \quad (2.10)$$

Residual vector of observations for horizontal angle β_{jik} :

$$V_{\beta_{jik}} = a_{ij} \delta x_j + b_{ij} \delta y_j - a_{ik} \delta x_k - b_{ik} \delta y_k + (a_{ik} - a_{ij}) \delta x_{ji} + (b_{ik} - b_{ij}) \delta y_{ji} + l_{\beta} \quad (2.11)$$

Residual vector of observations for vertical angle Z_{ik} :

$$v_{Z_{ik}} = A_{ik} \delta x_i + B_{ik} \delta y_i + C_{ik} \delta z_i + A_{ki} \delta x_k + B_{ki} \delta y_k + C_{ki} \delta z_k + l_{Z_{ik}} \quad (2.12)$$

2.4. Application of spatial terrestrial geodetic - GNSS short distance networks in construction of works

It is suitable to apply the combined spatial terrestrial geodetic - GNSS networks short distance measurement networks in the construction of buildings with high accuracy and great height difference such as: construction of super-high-rise buildings, construction of hydroelectric power plants works. Especially in the work of staking out axis in super-high-rise buildings.

Conclusion of Chapter 2

Chapter 2 mentioned and analyzed the applicability of the short distance spatial terrestrial - GNSS networks in construction of works with the great height difference and high accuracy such as high-rise, super-high-rise buildings, hydropower plants. It classified measuring distance and determined suitable instrument for measuring the distance according to classifications. For short and very short distances, it is recommended to use TS while medium and long distances should be measured with GNSS.

Chapter 3

STUDYING THEORY OF ADJUSTMENT OF COMBINED SPATIAL TERRESTRIAL GEODETIC - GNSS NETWORKS SHORT DISTANCE NETWORKS APPLIED IN CONSTRUCTION SURVEYING

3.1. Coordinate systems and reference systems used to establish combined spatial terrestrial geodetic - GNSS networks

3.1.1. Coordinate systems in construction surveying

Geocentric coordinate system; geodetic coordinate system; local topocentric coordinate system; VN 2000 Vietnam national coordinate reference system; construction assumed coordinate system.

3.1.2. Selecting the coordinate and reference systems to establish spatial terrestrial - GNSS short distance networks applied in construction surveying

Within the research scope, local topocentric coordinate system is selected to adjust the combined spatial terrestrial - GNSS short distance networks. The selected origin is the central point of work; the level of the origin point is the average height of the constructing area or the 0.0-level of high-rise buildings.

3.2. The adjustment algorithm of combined spatial terrestrial - GNSS networks in a local topocentric coordinate system with one control point

3.2.1. Residual vector of observations of the combined spatial terrestrial -

GSNN short distance networks in local topocentric coordinate system

The residual vector of slope distance S_{ij} as (2.10), of horizontal angle β_{jik} as (2.11), of vertical angle Z_{ik} as (2.12), of GNSS measurements are as follows:

$$v_{\Delta X_{ij}} = -\delta x_i + \delta x_j + l_{\Delta X_{ij}}; \quad v_{\Delta Y_{ij}} = -\delta y_i + \delta y_j + l_{\Delta Y_{ij}}; \quad v_{\Delta Z_{ij}} = -\delta z_i + \delta z_j + l_{\Delta Z_{ij}} \quad (3.1)$$

3.2.2. Adjustment procedure of combined spatial terrestrial geodetic - GNSS networks in the local topocentric coordinate system of one base point

Steps of adjusting the spatial terrestrial geodetic - GNSS networks in the local topocentric coordinate system of one base point are as follows [16]:

Step 1: Change coordinates of origin point to the geocentric coordinates B_G, L_G .

Step 2: Establish the rotation matrix R based on B_G, L_G .

$$R = \begin{bmatrix} -\sin B_G \cos L_G & -\sin L_G & \cos B_G \cos L_G \\ -\sin B_G \sin L_G & \cos L_G & \cos B_G \sin L_G \\ \cos B_G & 0 & \sin B_G \end{bmatrix} \quad (3.2)$$

Step 3: Calculate the transformation of factors $\Delta X, \Delta Y, \Delta Z$, the covariance matrix C_{XYZ} in geocentric coordinates to n factors $\Delta x, \Delta y, \Delta z$, the covariance matrix variance M in topocentric coordinates, based on the rotated matrix R .

Step 4: Test misclosure of f_x, f_y, f_z

Step 5: The first adjustment only includes the GNSS measurements by the parametric adjustment method. The obtained coordinates after the first adjustment of the points and posterior variance μ_{GPS} .

Step 6: The second adjustment combines GNSS measurements with angle β and slope distance S . Coordinates of x, y, z of the first adjustment is considered as approximate coordinates. The weights of the GNSS measurements:

$$\tilde{P}_{GPS} = \frac{1}{\mu_{GPS}^2} M_{xyz}^{-1} \quad (3.6)$$

Step 7: Calculate coordinates to assumed topocentric coordinates system.

3.2.3. Calculate local topocentric coordinates transformation to assumed topocentric coordinates system

Coordinates of points after adjustment in a local topocentric coordinate system $((x, y, z))$ must be transferred to a assumed topocentric coordinates system of works (x', y', z') .

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x + C_x \\ y + C_y \\ z \end{bmatrix} \quad (3.18)$$

3.3. Combined adjustment of spatial terrestrial geodetic - GNSS networks according to free adjustment with defect algorithm

The research scope of the thesis is applied in construction. The one base point networks show some limitations in multiple cycles measurement. In many cases, especially with the geodetic networks used for a long period of construction, the networks are evaluated better if they are treated by free adjustment with defect algorithm. Therefore, the free adjustment with defect algorithm ($d = 3$) is proposed to process data of the combined spatial terrestrial geodetic - GNSS networks in construction. Solutions to the standard system of equations with singular R matrix require the added system of d equations.

$$C^T X + L_C = 0 \quad (3.19)$$

R^{-1} the general matrix inversion that can be calculated by the formula:

$$R^{-1} = (R + C\bar{P}C^T)^{-1} - T\bar{P}T^T \quad (3.22)$$

in which: $C_i = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ with points to be used as positioning points.

$C_i = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ with points not to be used as positioning points

Other steps follow steps in the one base point network adjustment.

3.4. Algorithm of detecting systematic errors in vertical angle measurement

With the high precision of today's modern TS, and the vertical angle measurement errors of $\pm 1''$ or smaller, it is possible to measure vertical angles in spatial terrestrial - GNSS networks. The measurement of vertical angles is influenced by many systematic errors, mainly the refractive ones. In order to improve the usability of this measurement and to increase the rigidity of spatial networks in construction, the algorithm of detecting systematic errors of vertical angle measurements is proposed. Starting from the conditional adjustment with unknown numbers, a new unknown number is put in residual vector of observations of vertical angle measurements, being the systematic error of its, denoted by x . Results will determine the systematic error x . In the scope of works with similar distance, the residual vector of observations with unknown number x is the systematic error with the vertical angle Z_{ik} :

$$v_{Z_{ik}} = A_{ik}\delta x_i + B_{ik}\delta y_i + C_{ik}\delta z_i + A_{ki}\delta x_k + B_{ki}\delta y_k + C_{ki}\delta z_k + x + l_{Z_{ik}} \quad (3.25)$$

In considering the effects of measuring the distance to systematic errors, the formula (3.25) is replaced by the formula (3.26).

$$v_{Z_{ik}} = A_{ik}\delta x_i + B_{ik}\delta y_i + C_{ik}\delta z_i + A_{ki}\delta x_k + B_{ki}\delta y_k + C_{ki}\delta z_k + D_{ik}x_{Z_{ik}} + l_{Z_{ik}} \quad (3.26)$$

3.5. Establishing adjustment software of spatial terrestrial-GNSS networks

The function of the program is to process data of spatial terrestrial - GNSS networks, analyze the stability of positioning networks and calculate coordinate transformation. The calculations from AdNet2.0 software have been verified and compared to manual calculations, giving matching results.

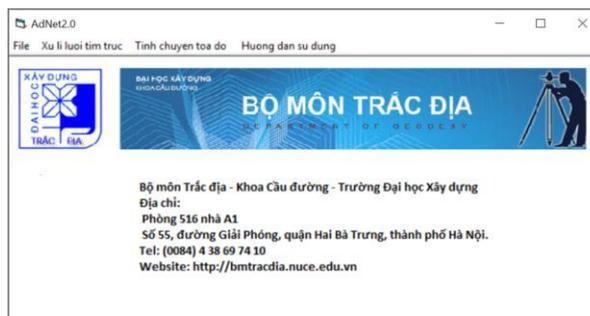


Figure 3.2. Interface of AdNet2.0 program

3.6. Experiment to process data of combined spatial terrestrial geodetic - GNSS short distance networks in construction with AdNet2.0 software

3.6.1. Introduction of works and experimental networks

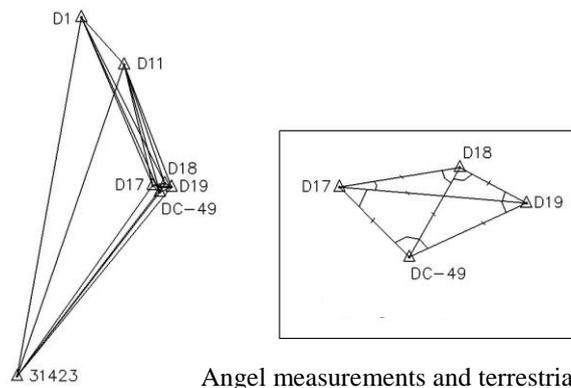


Figure 3.5. Diagram of control networks for monitoring and determining reservoir sedimentation of Nho Que 3 hydroelectric power plant

Current surveying tasks at Nho Que 3 hydroelectric plant is to monitoring and determine the sedimentation of the reservoir. Control networks consists of 7 points, located on three high levels of 1,200m, 350m and 300m. All points are measured using GPS. Particularly, 4 points in the area of the dam route D17, D18, D19, DC-49 are additionally measured with terrestrial measurements (8 angles and 6 distances) by TS.

3.6.2. Results of processing spatial terrestrial geodetic - GNSS networks

The networks have a significant the height difference of points measurements can combined terrestrial geodetic and GNSS measurements. The spatial terrestrial - GNSS networks are adjusted with one base point adjustment method by AdNet2.0. The results meet required accuracy of the works.

Table 3.7. Typical parameters of network accuracy

Indicator	Value
Posterior variance	0.938
Weakest point: D1	0.002 m
Length of weak distance: D18 - D19	1/90900
Azimuth of weak distance D18 - D19	$m_{\alpha} = 0.94''$

Conclusion of chapter 3

1- Chapter 3 presents steps for processing the data of spatial terrestrial geodetic - GNSS networks, in which the free adjustment with the defect is proposed to process data of this network in construction. The topocentric coordinate system is used as an intermediate coordinate system to perform the spatial adjustment problem, then transferred to the coordinate system of works.

2- The theory of the algorithm for detecting the value of systematic error existing in vertical angle measurement is presented in Chapter 3. The algorithm is combined in processing data of spatial terrestrial geodetic - GNSS networks.

3- AdNet2.0 software is built to serve the purpose of processing data of spatial terrestrial geodetic - GNSS networks.

Chapter 4

RESEARCHING METHODS FOR PROCESSING COMBINED SPATIAL TERRESTRIAL GEODETIC - GNSS NETWORKS SHORT DISTANCE NETWORKS APPLIED IN STAKING OUT AXIS AND TRANSFERRING HEIGHT IN SUPER-HIGH-RISE BUILDINGS

4.1. Application of spatial terrestrial - GNSS short distance networks in staking out axis in construction of super-high-rise buildings

4.1.1. The work of staking out axis in super-high-rise building

To perform the work of staking out axis in high-rise buildings, super-high-rise building, vertical collimator and segment method (each segment of $10 \div 15$ floors) can be used. To minimize accumulative errors, it is necessary to do re-

measurement the control networks at each end of the segment with combined TS and GNSS on floors.

4.1.2. The required accuracy of control networks for staking out axis in construction of super-high-rise buildings

1. Required accuracy for staking out axis and positioning networks at 0.0 level

GNSS combines two order networks (positioning and staking-out-axis networks) into a unified level. Standard deviations of one-level terrestrial - GNSS network to staking out axis at 0.0 level should $\leq \pm 2$ mm.

2. Required accuracy for networks on the first floor of each projection segment

Standard deviations of network on first floor of each projection segment:

$$m_{kc}^i \leq \pm \frac{\overset{CP}{\Delta_{XD}}}{7} \approx \pm \frac{H_i}{7000} \text{ (mm)} \quad (4.2)$$

4.1.3. The process of staking out axis in construction of the super-high-rise building with total station and GNSS

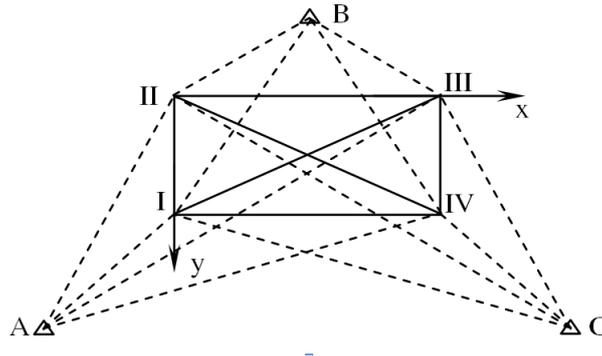


Figure 4.2: The system of combined terrestrial geodetic - GNSS control network in the construction of the super-high-rise building

Control network for staking out axis in super-high-rise buildings includes two types: positioning network and staking-out-axis network. Both are connected by GNSS. Staking out axis network is additionally measured by TS.

4.1.4. Designing positioning network and staking-out-axis network

At 0.0 level, it is required to estimate the accuracy of the network for staking out axis as the combined spatial terrestrial - GNSS networks.

4.1.5. Measure and process results of spatial terrestrial - GNSS networks applied to stake out axis in construction of super-high-rise buildings

Data of combined spatial terrestrial geodetic - GNSS networks at 0.0 level or on the first floor of each projection segment must be processed by adjustment of 3D free network algorithm with defect in topocentric coordinate system. After

adjustment, Helmert must be calculated the transformation to conventional topocentric coordinate system to match the works.

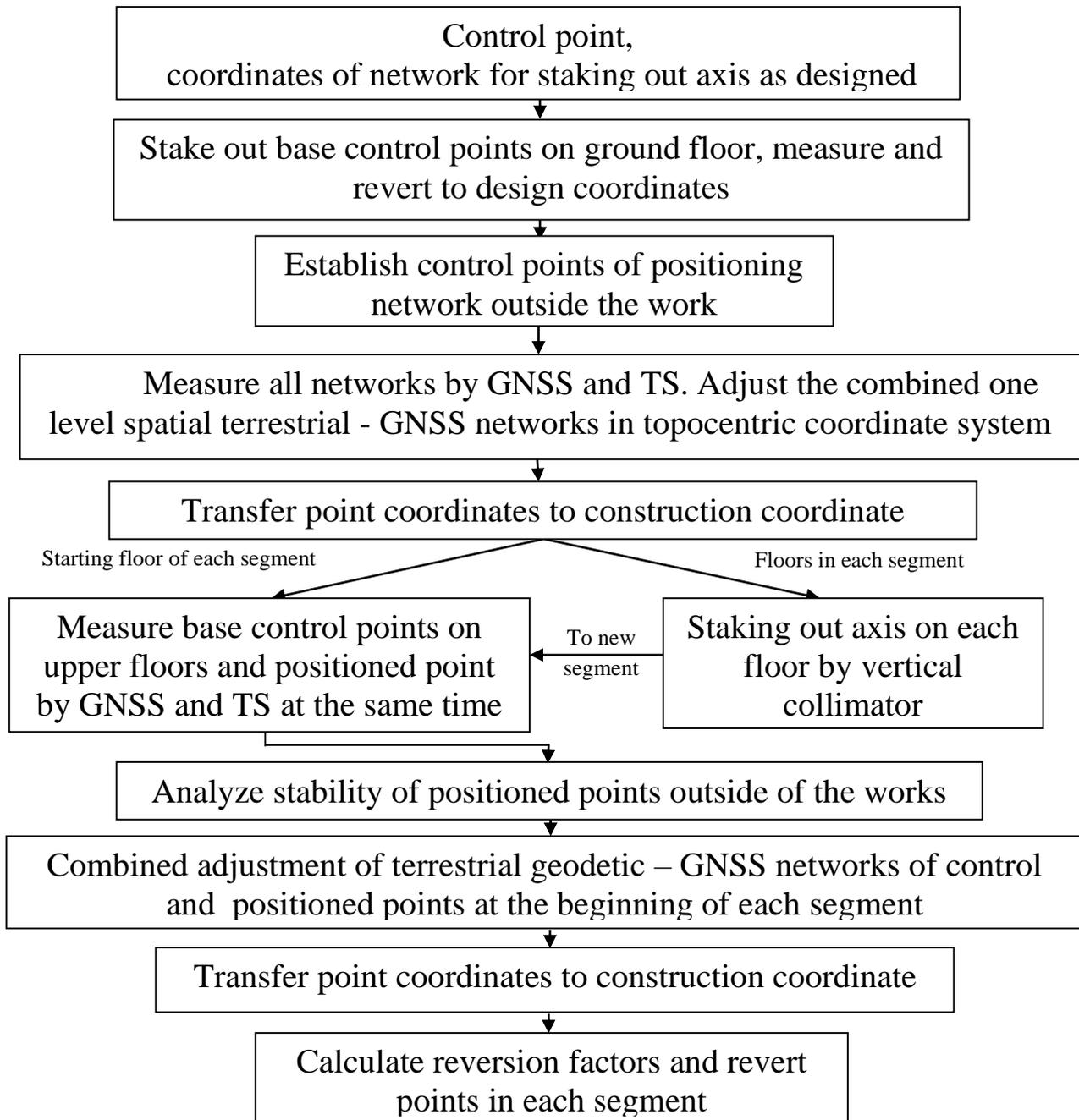


Figure 4.3. Diagram of steps for staking out axis in the super-high-rise building with the use of total station and GNSS

4.2. Calibrate measurement in short-distance spatial network before adjust

4.2.1. Calibrate effects of the instrument, mirrors height to measuring distance by TS and GNSS

GNSS measurements have been calibrated with the height of GNSS receiver. Calculating transformation of distance measurement by TS to control point, mirror center:

Effects of plumb deviations by height of works in establishing networks for staking out axis in super-high-rise buildings by terrestrial and GNSS are considerable and should be calibrated, particularly in more than 100m high.

Table 4.1. Effects of plumb deviations in Cartesian coordinates of points by height of works

Height (m) \ Plumb deviation (")	50	100	150	200	250	300	350	400	450	500
2	0.5	1.0	1.5	1.9	2.4	2.9	3.4	3.9	4.4	4.8
4	1.0	1.9	2.9	3.9	4.8	5.8	6.8	7.8	8.7	9.7
6	1.5	2.9	4.4	5.8	7.3	8.7	10.2	11.6	13.1	14.5
8	1.9	3.9	5.8	7.8	9.7	11.6	13.6	15.5	17.5	19.4
10	2.4	4.8	7.3	9.7	12.1	14.5	17.0	19.4	21.8	24.2
12	2.9	5.8	8.7	11.6	14.5	17.5	20.4	23.3	26.2	29.1
15	3.6	7.3	10.9	14.5	18.2	21.8	25.5	29.1	32.7	36.4

4.3.2. Effects of parallel misalignment between plumb lines passing through points

Table 4.2. Effects of parallel misalignment between plumb lines to distance ΔS_H (mm) by height of projection plane

$\Delta H(m)$ \ S (m)	50	75	100	150	200	300	400
25	0,20	0,29	0,39	0,59	0,78	1,18	1,57
50	0,39	0,59	0,78	1,18	1,57	2,35	3,14
75	0,59	0,88	1,18	1,77	2,35	3,53	4,71
100	0,78	1,18	1,57	2,35	3,14	4,71	6,28

Due to parallel misalignment between the plumb lines passing through the beginning and endpoints of the distance, any distance in network on the floor, when corrected to the original plane, will generate a deviation value ΔS_H :

$$\Delta S_H = -\frac{\Delta H}{R_m} \cdot S \quad (4.17)$$

Calibrations coordinate of the beginning and endpoints of the diagonal lines:

$$v_{\delta x} = \Delta S_H \cos \alpha ; v_{\delta y} = \Delta S_H \sin \alpha \quad (4.19)$$

It is realized that vertical projection distance when staking out coordinates on the floor is affected by the parallel misalignment of the plumb lines.

4.3.3. Solutions to determine plumb deviations in high-rise buildings

In small area of high-rise building, plumb deviation can be determined:

Step 1: The minimum and a most reasonable number of positioning control points is 3. Measure and connect a positioning point (denoted by point A) with one point in coordinates national system by GNSS. Determine the geoid height at A in one of two ways: leveling orthometric height or interpolation of geoid height from EGM-2008 [6].

Step 2: Use GNSS between positioning points to determine the ellipsoid height, and at the same time measure the exact geometrical leveling (level II). Determine the geoid height of the remaining positioning control points.

Step 3: Present the geoid height as a linear equations of the geodetic coordinates B, L. a_0, a_1, a_2 are found.

$$\zeta_i = a_0 + a_1B_i + a_2L_i \quad (4.26)$$

Step 4: Calculate components of plumb deviations.

4.4. Solutions to determine the height by GNSS in spatial terrestrial geodetic - GNSS short distance networks

4.4.1. Accuracy required of staking out axis on construction floors in construction of super-high-rise buildings

According to [49] when $h > 30$ m, the standard deviations in staking out axis with the height h in construction of high-rise buildings $m_h \geq \pm 5$ mm.

4.4.2. Ability to meet the required accuracy for transferring height to floors by GNSS in construction of high-rise buildings

Table 4.4. Standard deviations of ellipsoid height difference $m_{\Delta H}$ according to the height of the projection point h

h (m)	100	200	300	400	500
$m_{\Delta H}$ (mm)	3.63	3.64	3.67	3.70	3.73

According to Table 4.4 on the standard deviations of ellipsoid height difference according to projection point height (with horizontal distance 300m from positioning point to base point on the foundation plan) and the relative static measurement accuracy of Trimble R7s, R8s, R9s, R10s, current GNSS

technology are capable of meeting the technical requirements of this work in the construction of high-rise buildings, especially super-high-rise buildings.

4.4.3. Algorithm for determining the orthometric height and accuracy when applying GNSS to transferring height to floor of super-high-rise construction

It is possible to determine orthometric height between I' on the floor at i^{th} measurement and I on the foundation plane at first measurement according to ellipsoid heights between I' and I with A and subsidence of point I:

$$(h_{I'}^{(i)} - h_I^{(i)}) = (\Delta H_{AI'}^{(i)} - \Delta H_{AI}^{(0)}) - s_I^{(i)} \quad (4.42)$$

Value $m_{s_I} = \pm (1 \div 1.5)$ mm when subsidence monitoring with bored pile foundation has negligible effects compared to other effects. For $m_{\Delta H} = \pm 3.6$ mm (table 4.5), it is calculated as $m_{\Delta h_{i-i'}} = \pm 5.1$ mm. This accuracy meets required accuracy in transferring height to upper floors of super-high-rise, with $h > 30$ m.

4.5. The experiment of measuring and processing data of combined spatial terrestrial geodetic - GNSS short distance networks to stake out axis in construction of super-high-rise buildings

4.5.1. Introduction of experimental works and network measurement results

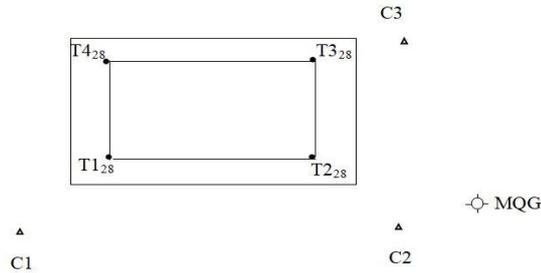


Figure 4.8. Diagram of control network on 28th floor and positioning network

Construct spatial terrestrial - GNSS networks for staking out axis to the 28-storey CT2 building, being the Army residence, Thach Ban, Hanoi. The 7-point network includes: 4 points for staking out axis on the 28th floor (T1, T2, T3, T4); 3 positioning points (C1, C2, C3) at stable positions on the ground. All 7 points are measured by GPS. 4 points T1, T2, T3, T4 are measured by TS, 8 angles, 6 distances. Data is processed to find out the coordinates of 7 points in a coordinate system consistent with the base network at 0.0 level.

4.5.2. Processing actual measurement data of networks

The experimental networks with actual measured data are considered the first cycle and processed according to the following plans:

- *Plan 1*: Free adjustment of quadrilateral geodetic networks $T_1T_2T_3T_4$ on floors.
- *Plan 2*: Adjustment of spatial terrestrial geodetic - GNSS networks (7 points) in the topocentric coordinate system according to one base point plan (C_2).
- *Plan 3*: Adjustment of spatial terrestrial geodetic - GNSS network (7 points) in the topocentric coordinates system according to free adjustment with defect plan (position 3 points C_1, C_2, C_3).

Comment: The coordinates of points and distances of adjustment between calculation plans have the same results. The reason is that the plans have the same measurement data and are not affected by the base point error. This also proves that the measurements are of equal and high accuracy.

Table 4.6. Distances of networks after adjustment in calculation methods (m)

Distance	Adjustment of quadrilateral networks (PA1)	Adjustment of GPS – TS networks with fixed point (PA2)	Free adjustment of GPS – TS (PA3)	Different among plans
$T_1 - T_2$	17,158	17,158	17,158	0,000
$T_1 - T_3$	33,853	33,853	33,853	0,000
$T_1 - T_4$	30,653	30,653	30,653	0,000
$T_2 - T_3$	20,069	20,069	20,069	0,000
$T_2 - T_4$	24,817	24,817	24,817	0,000
$T_3 - T_4$	15,535	15,535	15,535	0,000

4.5.3. Process data of assumed networks in the displacement of point C1

Results of free adjustment of spatial terrestrial - GNSS networks (option 3) is used as the basis, which is considered as the result in the first cycle. In the second cycle, due to no measurement data, it is assumed to transfer geocentric coordinate of point C1 with displacement values: $\delta X = - 7$ mm; $\delta Y = + 1$ mm; $\delta Z = - 3$ mm. Recalculate baseline GNSS as assumed measurement data in second cycle. Coordinate difference of point C1 between 2 cycles in topocentric coordinate system is calculated as: $\delta x = - 4$ mm; $\delta y = + 6$ mm; $\delta z = 0$ mm.

Assumed measurements are processed by the following three plans:

Option 1: Parametric adjustment of spatial terrestrial - GNSS networks in the topo-centric coordinate system with one base point C2 (no displacement of point)

Table 4.9. Coordinate difference of points between 1st and 2nd cycles (unit mm)

Point	δx	δy	δz	Difference between 2 cycles
T ₁	0	0	0	0.0
T ₂	0	0	0	0.0
T ₃	0	0	0	0.0
T ₄	0	0	0	0.0
C1	-1	+6	-4	7.8
C3	0	0	0	0.0
C2	0	0	0	0.0

Comment: The topocentric coordinate difference of C1 is different from the assumption. Adjustment spatial terrestrial geodetic - GNSS networks with one base point fails to accurately detect the displacement of the control point.

Option 2: Parametric adjustment of spatial terrestrial - GNSS networks in topocentric coordinate system with one base point C2 (with displacement of point)

Table 4.11. Coordinate difference of points between 1st and 2nd cycles (unit mm)

Point	δx	δy	δz	Difference between 2 cycles
T ₁	0	-5	+4	6.4
T ₂	+1	-5	+4	6.5
T ₃	0	-6	+4	7.2
T ₄	0	-5	+5	7.1
C1	0	0	0	0.0
C3	+2	-5	+2	5.7
C2	+1	-5	+3	5.9

Comment: The topocentric coordinate difference of C1 is different from the assumption. Adjustment spatial terrestrial geodetic - GNSS networks with one base point fails to accurately detect the displacement of the control point. Especially, when the displaced point is chosen as the origin, it is dangerous because result will give erroneous values at all points coordinates in network.

Plan 3: Adjustment of the spatial terrestrial geodetic - GNSS networks in the topocentric coordinate system by the free adjustment with the defect.

To solve a singular R matrix, it is necessary to calculate \tilde{R} by the formula (3.25) with the matrix C_i selected as follows:

In which: $C_i = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ with three positioning points C1, C2, C3.

$C_i = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ with points not to be used as positioning points.

Comment:

1. The coordinate difference of C1 is exactly equal to the assumed value. Adjustment of spatial terrestrial - GNSS networks by the free adjustment with defect algorithm detect the exact displacement of the control point.
2. The coordinate difference after adjustment of the remaining points between two cycles is zero, because there is no displacement of these points according to the initial assumption.

Table 4.13. Coordinate difference of points between 1st and 2nd cycles (unit mm)

Point	δx	δy	δz	Difference between 2 cycles
T ₁	0	0	0	0.0
T ₂	0	0	0	0.0
T ₃	0	0	0	0.0
T ₄	0	0	0	0.0
C1	-4	+6	0	7.2
C3	0	0	0	0.0
C2	0	0	0	0.0

4.5.4. Assess free adjustment with defect algorithm to process data of spatial terrestrial - GNSS short distance networks in staking out axis of super-high-rise buildings

The calculations of the above plans also show that the study of free adjustment algorithms for spatial terrestrial geodetic - GNSS networks in both theory and practice has been successfully implemented to process data of the network at the beginning of each projection segment in super-high-rise.

4.5.5. Evaluate the ability to meet the required accuracy for transferring height to floors by GNSS in construction of high-rise buildings

From results of processing data of plan 3 in Section 4.5.2, the coordinate standard deviations z (topographic elevation) of points in the experimental network has the maximum value of ± 3 mm (Table 4.14).

Table 4.14. Topographic coordinates and standard deviations of points (m)

Points	x	m _x	y	m _y	z	m _z
T ₁	2325318.371	0,001	593982.487	0,001	101,235	0,001
T ₂	2325321.594	0,001	593999.340	0,001	101,236	0,003
T ₃	2325339.000	0,001	594009.330	0,001	101,222	0,001
T ₄	2325346.116	0,001	593995.520	0,001	101,215	0,002
C1	2325238.228	0,001	593780.240	0,001	17,469	0,002
C3	2325214.224	0,001	594091.667	0,001	11,554	0,002
C2	2325142.715	0,001	593812.068	0,001	17,571	0,002

At the same time, the trigonometric height difference on the 3 C1 - T₁, C2 - T₁ and C1 - T₄ by Leica TC-1201 TS are compared to geodetic difference in spatial terrestrial geodetic - GNSS networks ($\Delta H_{\text{GPS-MD}}$).

Table 4.15. Compare trigonometric height difference and geodetic difference

No	Starting point	Ending point	Δh_{TDDT} (m)	$\Delta H_{\text{GPS-MD}}$ (m)	δ_h (mm)
1	C ₁	T ₁	83,761	83,765	+ 4
2	C ₂	T ₁	83,659	83,663	+ 4
3	C ₁	T ₄	83,743	83,746	+ 3

Comments: Differences of δ_h have small values, which is suitable with profile of total station and GPS as well as results of network adjustments. The above results have confirmed the applicability of GNSS technology to stake out axis in the construction of super-high-rise buildings.

4.5.6. Experiment calculation and detection of systematic errors in measuring vertical angle of the combined spatial terrestrial geodetic - GNSS networks

a. Experiment with as the average value from multiple measurements data

Network diagram is used to and measurement content is as in section 4.5.1, in addition measurement of 3 more slope angles. These slope angles are measured many times, at different times of the day and the average value is selected. With relatively uniform distances, the residual vector of vertical angle as formula (3.25). Unknown number $x = -0.5''$ is detected by calculation results. Model data are selected and calculations with 9 plans are made (table 4.17). A difference of $\leq \pm 0.5''$ of systematic error x is found compared to the assumed value. It proves correctness of the systematic error of vertical angle measurement detection

algorithm combined in processing data of the spatial terrestrial geodetic - GNSS networks .

Table 4.17. Result of calculating systematic error of slope angle

No	Plan	Assumed systematic error	Assumed random error	Gained result of systematic error (x)
1	Initial SL	0	0	$x = -0.5''$
2	Plan 1	$+1''$	0	$x = +0.5''$
3	Plan 2	$-2''$	0	$x = -2.5''$
4	Plan 3	$+3''$	0	$x = +2.8''$
5	Plan 4	$-6''$	0	$x = -6.2''$
6	Plan 5	$+8''$	0	$x = +7.5''$
7	Plan 6	$+12''$	0	$x = +11.5''$
8	Plan 7	$-2''$	$\pm 1''$	$x = -2.2''$
9	Plan 8	$-3''$	$\pm 3''$	$x = -3.7''$
10	Plan 9	$+8''$	$\pm 2''$	$x = +7.4''$

b. Experiment with measurement data at different times of the day

With 08:30 a.m measurement, the systematic error of vertical angle is not significant with $x = -0.31''$. Calculations at 11:45 a.m and 14:45 p.m have considerable systematic errors of the vertical angle of $x = -4.40''$ and $x = -2.97''$, respectively. Therefore, appropriate time of measurement should be chosen to avoid refractive errors.

4.5.7. Experiment processing data of spatial terrestrial - GNSS short distance networks in inclined deformation monitoring of high-rise buildings

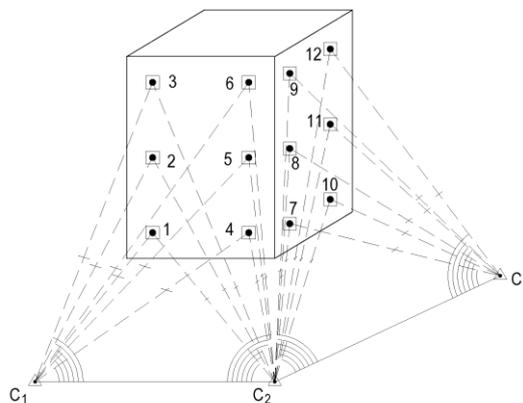


Figure 4.9. Diagram of experimental spatial short distance network in slope deformation of high-rise buildings

For same location with experiment introduced in Section 4.5.1, use TS and GPS to determine slope deformation of work. The base control point network includes 3 points: C1, C2, C3 are established from the experiment of staking out axis. Inclined deformation monitoring points include 12 points mounted on 2 building surfaces. At each monitoring point, distance, horizontal angle and vertical angle are measured several times, the average value selected. One level adjustment of combined spatial terrestrial geodetic - GNSS networks short distance is applied, coordinates of the points are gained and slope is determined.

Table 4.23. Results of determining slope deformation of axes

Straight line	e_x (m)	e_y (m)	e (m)	Height	Vertical angle
1-2	0,025	-0,05	0,026	34,931	0°02'33''
1-3	0,044	-0,028	0,052	66,934	0°02'40''
4-5	0,009	-0,012	0,015	38,366	0°01'21''
4-6	0,010	-0,027	0,029	70,387	0°01'25''
7-8	0,000	-0,015	0,015	38,395	0°01'21''
7-9	-0,020	-0,021	0,021	71,057	0°01'01''
10-11	0,015	0,032	0,035	38,380	0°03'08''
10-12	0,008	0,014	0,016	70,396	0°00'47''

Comments: With typical feature of positions for slope deformation as being unable to place an ordinary TS, theodolite or mirror, in order to measure the slope deformation of building, it is necessary to build a GPS base network system and deformation points by mirror-less TS with high accuracy. The combined spatial terrestrial - GNSS short distance networks in one-level is adjusted to achieve better accuracy.

Conclusion of Chapter 4

1. In order to accuratize at the beginning of each projection segment network in the formula of staking out axis in high-rise buildings, it is necessary to combine terrestrial and GNSS measurements. Chapter 4 presents the process of staking out axis in high-rise buildings with the application of TS and GNSS. The reasonable plan to process the data is the free adjustment algorithm of spatial terrestrial geodetic - GNSS networks.

2. The reference system of topocentric coordinate system and Cartesian coordinate system is different. By the height of building, this difference will affect

to staking out point of axis to the working platforms. It is necessary to calibrate into the work coordinates x' , y' on the segments affected by the plumb line deflection, the parallel misalignment of plumb lines, to ensure the construction axis to be always vertical along the plumb line.

3. It is possible to apply GNSS technology to transferring height to floors in the construction of super-high-rise buildings. The advantage of the method is that the accuracy of determining the height difference between the points on the floor and the foundation plane point is almost independent of the projection point height.

4. The content of experimental measurements and calculations aims to re-confirm the correctness of theoretical studies: The ability to apply spatial terrestrial geodetic - GNSS networks in staking out axis and transferring height to floors in super-high-rise buildings; Advantages of free adjustment algorithm in processing data of combined spatial terrestrial geodetic - GNSS networks; The algorithm to detect the systematic error in measuring the vertical angle of spatial terrestrial geodetic - GNSS networks gives the correct results as assumed; It is possible to apply TS and GNSS together with the algorithm to process combined spatial terrestrial geodetic - GNSS networks in the slope deformation of super-high buildings.

CONCLUSION AND RECOMMENDATIONS

CONCLUSION

From researches, surveys and analysis of the theory and experiments in calculating and processing data of the combined spatial terrestrial geodetic - GNSS short distance networks in construction, some conclusions are presented as the followings:

1. In some cases, it is advisable to apply the combined spatial terrestrial geodetic - GNSS short distance networks in construction of buildings with high accuracy such as: construction of super-high-rise buildings, construction of hydroelectric power plants works, especially in the work of staking out axis in super-high-rise buildings.
2. To accuratize axis network at the beginning of each projection segment in the construction of super-high-rise buildings, it is recommended to apply the method of free adjustment with defects of spatial network including the measurements of GNSS and TS with at least three positioning points.

3. To ensure the accuracy of staking out axis in super-high-rise buildings, it is necessary to correct the measured values before adjustment and calibrate the coordinates of points after adjustment, in which it is necessary to pay attention to vectors affected by plumb line deflection and the parallel misalignment of plumb lines along with the height of works, to ensure that the building axis is always vertical.
4. It is possible to apply GNSS technology to transferring height to upper floors in construction of high-rise buildings, this solution ensures the required accuracy and has many efficiency when combined with staking out axis using TS and GNSS.
5. The algorithm for detecting systematic error in the results of vertical angle measurement combined in processing data of the spatial terrestrial geodetic - GNSS networks gives consistent results and can be applied in practice to improve results after adjustment. When measuring the vertical angle of the day, it is necessary to choose the appropriate measurement times to avoid the effect of systematic error, which is mainly due to refraction.
6. AdNet2.0 software completely meets the requirements for processing data of combined spatial terrestrial geodetic - GNSS networks short distance networks.

RECOMMENDATIONS

After the process of researching theory, participating in actual production and implementing the thesis, some recommendations are suggested as follows:

1. It is necessary to have further researches and efficiently exploit the types of modern measuring equipment and technology for continuous measurement in order to improve the efficiency of staking out axis in super-high-rise buildings. Especially, it is advised to research the effect of the building's vibration in relation to the measurement results on the floors with great high difference.
2. It is recommended to apply the method of free adjustment with defects when processing data of combined spatial terrestrial geodetic - GNSS networks in construction of works.
3. It is necessary to issue "Standards for surveying work in construction of super-high-rise buildings" as a basis for production, when this type of work is being built more and more and the thesis needs study and clarify some necessary contents.

**LIST OF SCIENTIFIC WORKS PUBLISHED BY AUTHORS
RELATING TO THE THESIS CONTENTS**

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